P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II B.Sc. MATHEMATICS-Semester IV (w.e.f. 2017-2018)
Course: Real Analysis

Total Hrs. of Teaching-Learning: 90 @ 6 hr/Week **Total credits: 5**

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.
- This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Unit I: Real Number System and Real Sequence **(18 hours)**

The algebraic and order properties of R – Absolute value and Real line – Completeness property of R – Applications of supreme property – Intervals - Limit point of a set - Existence of limit points. (No questions to be set from this portion)

Sequences and their limits – Range and Boundedness of sequences - Necessary and sufficient condition for convergence of Monotone sequence, limit point of a sequence, Subsequences and the Bolzano Weierstrass theorem - Cauchy sequences – Cauchy's general principle of convergence theorems.

Unit II: Infinite Series **(18 hours)**

Introduction to Infinite Series – Convergence of series – Cauchy's general principle of convergence for series – Tests for convergence of non negative terms – P - test – Limit comparision test – Cauchy's nth root test – De-Alambert's ratio test - Alternating series – Liebnitz's test - Absolute and conditional convergence.

Unit III: Limits and Continuity **(18 hours)**

Real valued functions – Boundedness of a function - Limit of a Function, One-sided Limits-Right hand and Left Hand Limits - Limits at Infinity - Infinite Limits. **(no question to be set)**

Continuous Functions - Discontinuity of a Function - Algebra of Continuous Functions – Continuous functions on intervals - Some Properties of Continuity of a function at a point - Uniform Continuity.

Unit IV: Differentiation and Mean Value Theorem **(18 hours)**

The Derivability of a function, on an interval, at appoint, Derivability and Continuity of a function - Geometrical meaning of the Derivative - Mean Value Theorems - Rolle's Theorem, Lagrange's Mean Value theorem, Cauchy's Mean Value theorem.

Unit V: Riemann Integration **(18 hours)**

Riemann sums, Upper and Lower Riemann integrals, Riemann integral, Riemann Integrable function – Darboux's Theorem - Necessary and sufficient conditions for Riemann integrability – properties of integrable functions – Fundamental Theorem of Integral Calculus – Integral as the limit of a sum – Mean Value Theorems.

Additional Inputs :

1. Problems using cauchy's first theorem on limits and cauchy's second theorem on limits.
2. Statement of Maclaurin's theorem and expansions of e^x, $\sin x$, $\cos x$, $\log(1+x)$.

Prescribed book:

- Real Analysis by Rabert & Bartely and D. R. Sherbart, published by John Wiley.

Reference books:

- Elements of Real Analysis by Santhi Nararayan & M. D. Raisinghania, published by S.Chand & Company Pvt. Ltd., New Delhi.
- Course on Real analysis by N. P. Bali - Golden series publications.
- A Text Book of Mathematics Semester IV by V. Venkateswarrao & others, published by S.Chand & Company Pvt. Ltd., New Delhi

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-IV

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted
I	Real Number System and Real Sequence	1	1	1	14
II	Infinite Series	1	1	2	22
III	Limits and Continuity	1	1	1	14
IV	Differentiation and Mean Value Theorem	1	1	2	22
V	Riemann Integration	1	1	2	22
	TOTAL	5	5	8	94

V.S.A.Q. = Very short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very short answer questions : 5 X 1 = 05
Short answer questions : 3 X 5 = 15
Essay questions : 5X 8 = 40

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Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Real Analysis
Paper–IV (Model Paper w. e. f. 2018-2019)

Time: 2Hrs 30 min | Max. Marks: 60

PART-I

Answer ALL the questions. Each question carries 1 mark. $5 \times 1 = 5$ M

1. Define convergence of a sequence.
2. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.
3. Define continuity of a function at a point.
4. Give an example of a function which is continuous but not derivable.
5. Evaluate $\lim_{n\to\infty} \frac{1}{n}\left[e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \cdots + e^{\frac{3n}{n}}\right]$.

PART -II

Answer any THREE questions. Each question carries 5 marks. $3 \times 5 = 15$ M

6. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \cdots + \sqrt{\frac{1}{n^2+n}}\right] = 1$.
7. State and Prove Leibnitz's test.
8. Examine for continuity the function f defined by $f(x) = |x| + |x-1|$ at 0 and 1.
9. Show that every derivable function on a closed interval is continuous.
10. State and prove fundamental theorem of Integral Calculus.

PART-III

Answer any FIVE questions from the following by choosing at least TWO from each section. $5 \times 8 = 40$ M
Each question carries 8 marks.

SECTION – A

11. Prove that every monotonically increasing sequence which is bounded above converges to its least upper bound.
12. State and Prove Cauchy's n^{th} root test for the convergence of series.
13. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$.
14. Prove that every continuous function is bounded and attains its bounds.

SECTION - B

15. State and prove Rolle's theorem.
16. Using Legrange's mean value theorem, prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$.
17. Prove that f(x) = sin x is integrable on $[0, \frac{\pi}{2}]$ and $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$.
18. State the first mean value theorem of integral calculus and by using it prove that $\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x}\, dx \le \frac{\pi^3}{6}$.